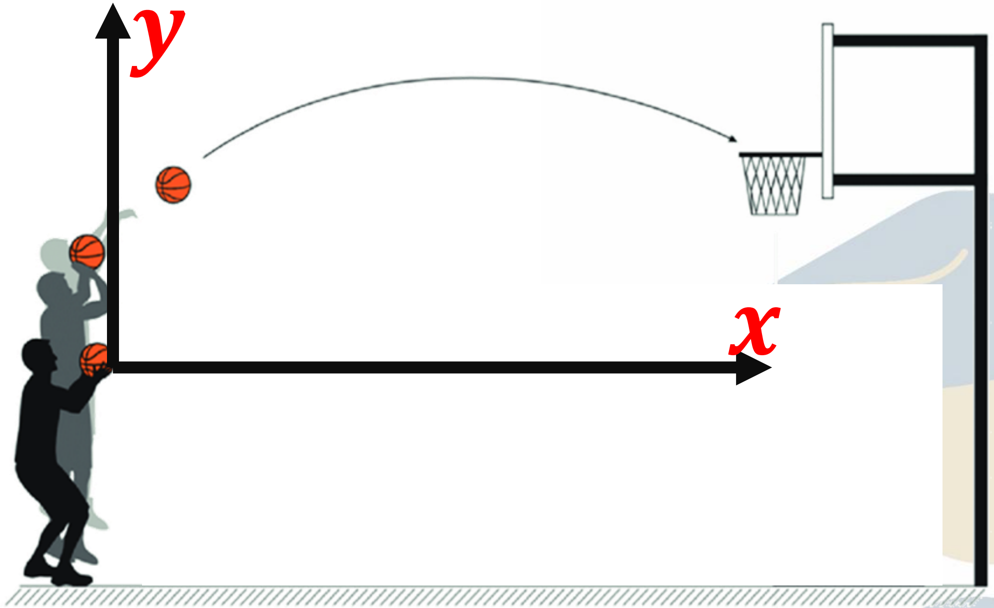


Physics - Grade 11 S

Unit Two: Mechanics

Chapter 7

Motion of a Particle in a Plane

Prepared & presented by : Mr. Mohamad Seif

Think then Solve

Exercise 1:

The motion of an object assimilated to a particle M in the plane of reference $(O, \vec{i}, \vec{j})$ is given by the position vector:

$$\vec{r}(t) = \overrightarrow{OM} = -3t\vec{i} + (t^2 - 1)\vec{j} \qquad \text{in SI}$$

1) Determine the velocity vector at any instant t and find its value at this instant.
2) Determine the acceleration vector at any instant t and find its value at this instant.
3) Determine the unit vector $\vec{u}_t$ in terms of $\vec{i}$ and $\vec{j}$.
4) Determine the tangential acceleration vector $\vec{a}_t$ in terms of $\vec{i}$ and $\vec{j}$ for $t = 0.5s$.

Exercise 1:

5) **Deduce the vector $\vec{a}_n$ in terms of $\vec{i}$ and $\vec{j}$ for $t = 0.5s$.**

6) **Calculate the value of $\vec{a}_n$ for $t = 0.5s$.**

Exercise 1:

$$\vec{r}(t) = \overrightarrow{OM} = -3t\vec{i} + (t^2 - 1)\vec{j}$$

1) Determine the velocity vector at any instant t and find its value at this instant.

The velocity vector ($\vec{V}$) is the derivative of position vector $\vec{r}$

$$\vec{V}(t) = \overrightarrow{r'}(\text{t}) = -3\vec{i} + 2t\vec{j}$$

The value of Velocity Vector is: $V = \sqrt{V_x^2 + V_y^2}$

$$V = \sqrt{(-3)^2 + (2t)^2}$$

$$V = \sqrt{9 + 4t^2}\ m/s$$

Exercise 1:

$\vec{r}(t) = \overrightarrow{OM} = -3t\vec{i} + (t^2 - 1)\vec{j}$; $\vec{V}(t) = -3\vec{i} + 2t\vec{j}$

2) Determine the acceleration vector at any instant t and find its value at this instant.

The acceleration vector ($\vec{a}$) is the derivative of velocity vector $\vec{V}$

$$\vec{a} = \vec{V'}(\mathrm{t}) = 2\vec{j}$$

The value of acceleration Vector is: $a = \sqrt{a_x^2 + a_y^2}$

$$a = \sqrt{(0)^2 + (2)^2}$$

$$a = 2m / s^2$$

Exercise 1:

$\vec{r}(t) = \overrightarrow{OM} = -3t\vec{i} + (t^2 - 1)\vec{j}; \vec{V}(t) = -3\vec{i} + 2t\vec{j}$

3)Determine the unit vector $\vec{u}_t$ in terms of $\vec{i}$ and $\vec{j}$.

$$\vec{V} = V.\vec{u}_t$$

$$\vec{u}_t = \frac{\vec{V}}{V}$$

$$\vec{u}_t = \frac{-3\vec{i} + 2t\vec{j}}{\sqrt{9 + 4t^2}}$$

Exercise 1:

$V = \sqrt{9+4t^2}$; $\vec{u}_T = \dfrac{-3\vec{i}+2t\vec{j}}{\sqrt{9+4t^2}}$

4) Determine the tangential acceleration vector $\vec{a}_t$ in terms of $\vec{i}$ and $\vec{j}$ for $t = 0.5s$.

$$\vec{a}_t = a_t.\vec{u}_T$$

$$a_t = V' = \frac{8t}{2\sqrt{9+4t^2}}$$

$$a_t = \frac{4t}{\sqrt{9+4t^2}}$$

$$\vec{a}_t = a_t.\vec{u}_T$$

$$\vec{a}_t = \frac{4t}{\sqrt{9+4t^2}}.\left(\frac{-3\vec{i}+2t\vec{j}}{\sqrt{9+4t^2}}\right)$$

$$\vec{a}_t = \frac{4t.(-3\vec{i}+2t\vec{j})}{9+4t^2}$$

Exercise 1:

$$\vec{a}_t = \frac{4t.(-3\vec{i} + 2t\vec{j})}{9 + 4t^2}$$

For $t = 0.5s$

$$\vec{a}_t = \frac{4(0.5)(-3\vec{i} + 2[0.5]\vec{j})}{9 + 4[0.5]^2}$$

$$\vec{a}_t = \frac{2(-3\vec{i} + 1\vec{j})}{9 + 1}$$

$$\vec{a}_t = \frac{-6\vec{i} + 2\vec{j}}{10}$$

$$\vec{a}_t = \frac{-3\vec{i} + \vec{j}}{5}$$

Exercise 1:

$\vec{a} = 2\vec{j}; \vec{a}_t = \frac{-3\vec{i}+\vec{j}}{5}$

5) Deduce the vector $\vec{a}_n$ in terms of $\vec{i}$ and $\vec{j}$ for $t = 0.5s$.

$\vec{a} = \vec{a}_t + \vec{a}_n \Rightarrow \vec{a}_n = \vec{a} - \vec{a}_t$

$$\vec{a}_n = 2\vec{j} - \left(\frac{-3\vec{i}+\vec{j}}{5}\right)$$

$$\vec{a}_n = \frac{2\vec{j}}{1} - \left(\frac{-3\vec{i}+\vec{j}}{5}\right)$$

$$\vec{a}_n = \frac{2\vec{j}\times 5}{1\times 5} - \left(\frac{-3\vec{i}+\vec{j}}{5}\right)$$

$$\vec{a}_n = \frac{2\vec{j}\times 5}{1\times 5} - \left(\frac{-3\vec{i}+\vec{j}}{5}\right)$$

$$\vec{a}_n = \frac{10\vec{j}}{5} + \frac{+3\vec{i}-\vec{j}}{5}$$

$$\vec{a}_n = \frac{3\vec{i}+9\vec{j}}{5}$$

Exercise 1:

$\vec{a} = 2\vec{j}; \vec{a}_t = \frac{-3\vec{i}+\vec{j}}{5}; \vec{a}_n = \frac{3\vec{i}+9\vec{j}}{5}$

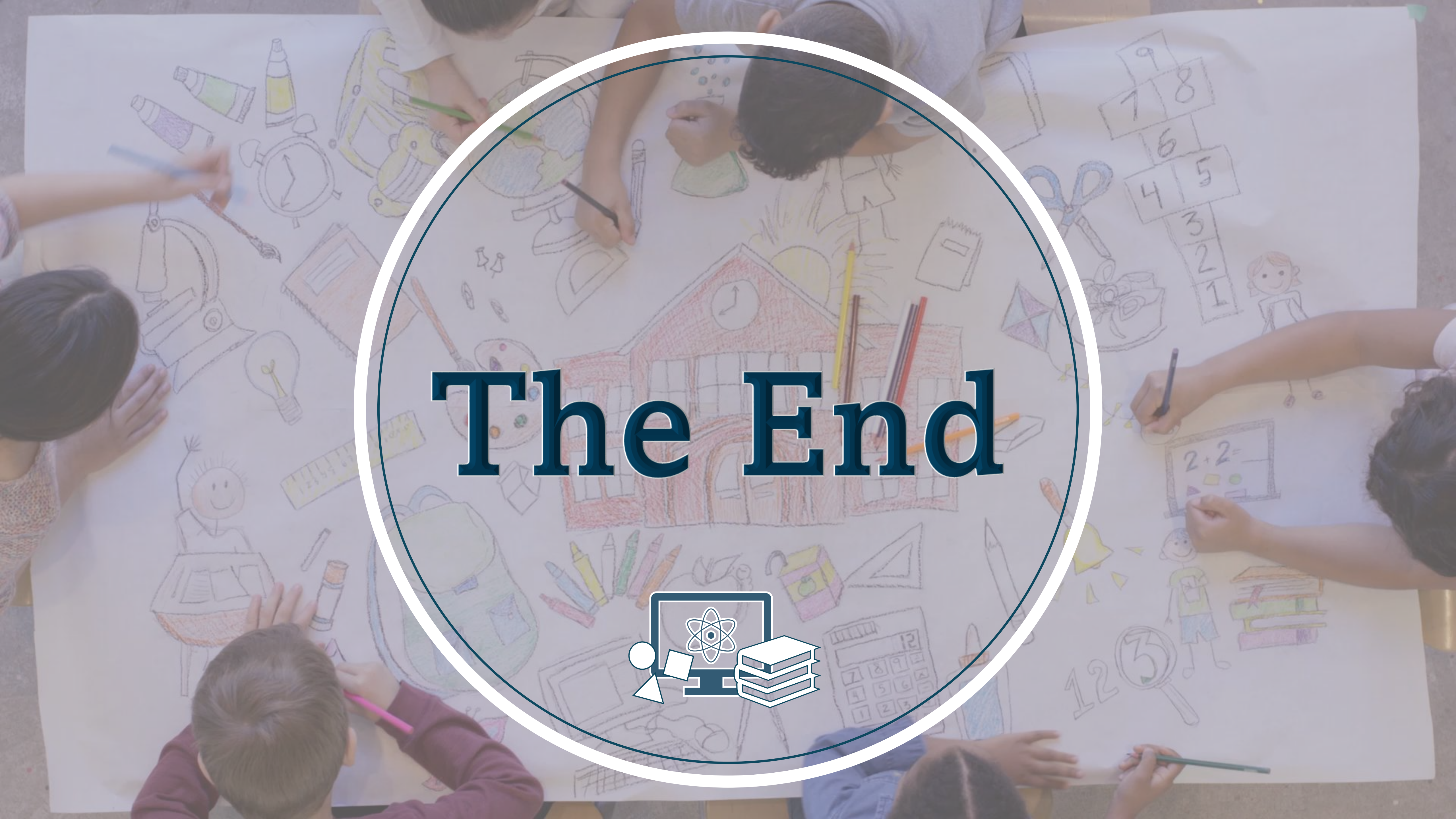
6) Calculate the value of $\vec{a}_n$ for $t = 0.5s$.

$$\vec{a}_n = \frac{3\vec{i} + 9\vec{j}}{5}$$

$$a_n = \sqrt{a_x^2 + a_y^2}$$

$$a_n = \sqrt{(\frac{3}{5})^2 + (\frac{9}{5})^2}$$

$$a_n = \sqrt{\frac{18}{5}} = 1.897\ m/s^2$$



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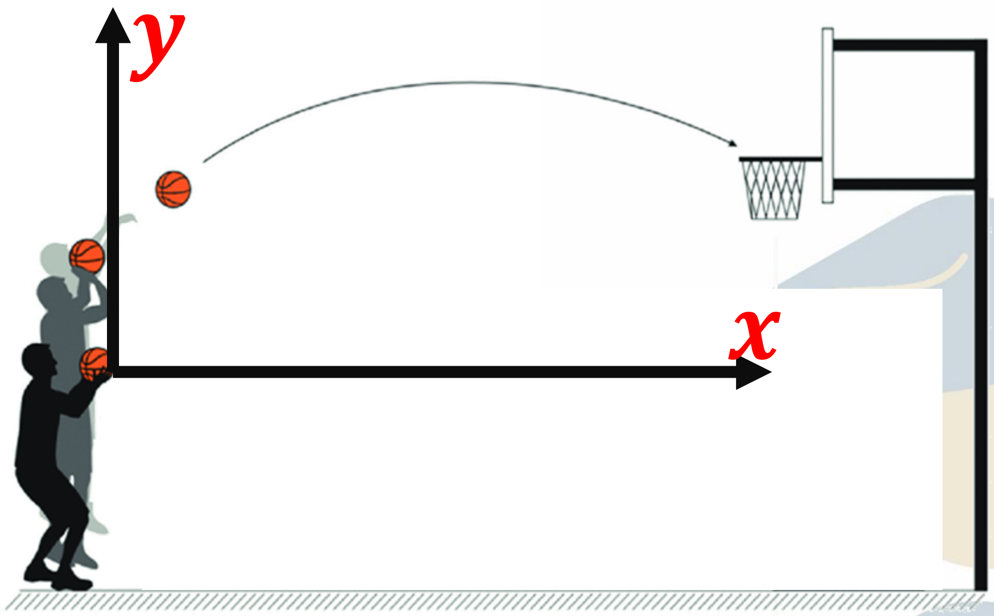


Physics - Grade 11 S

Unit Two: Mechanics

Chapter 7

Motion of a Particle in a Plane

Prepared & presented by : Mr. Mohamad Seif

Exercise 2:

A particle M is moving along a circle of radius 2m and such that its curvilinear position is given by:

$$s(t) = 4t^2 - t + 1 \qquad (m)$$

1) Determine the angular position of M at any instant and deduce its value at $t_0 = 0s$ and $t_1 = 1s$
2) Determine the angular speed at any instant t.
3) Determine the linear speed at any instant t using 2 methods
4) Determine the angular acceleration at any instant t
5) Calculate the linear acceleration at $t_1 = 1s$

Exercise 2:

$s(t) = 4t^2 - t + 1$

1) Determine the angular position of M at any instant and deduce its value at $t_0 = 0s$ and $t_1 = 1s$

$$s = R.\theta$$

$$\theta = \frac{s}{R} = \frac{4t^2 - t + 1}{2}$$

$$\theta = 2t^2 - 0.5t + 0.5 \text{ rd}$$

At $t_0 = 0$

$$\theta_0 = 2(0)^2 - 0.5(0) + 0.5 \text{ rd}$$

$$\theta_0 = 0.5 \text{ rd}$$

At $t_1 = 1s$

$$\theta_1 = 2(1)^2 - 0.5(1) + 0.5 \text{ rd}$$

$$\theta_1 = 2 \text{ rd}$$

Exercise 1:

$s(t) = 4t^2 - t + 1$; $\theta = 2t^2 - 0.5t + 0.5$ rd

2) Determine the angular speed at any instant t.

The angular speed is the derivative of angular position:

$$\omega = \theta' = 4t - 0.5\ (\text{rd}/s)$$

3) Determine the linear speed at any instant t using 2 methods.

$$V = s'(t) = 8t - 1\ (m/s)$$

Or

$$V = R.\theta' = 2(4t - 0.5)$$

$$V = 8t - 1\ (\text{m}/s)$$

Exercise 2:

$\theta = 2t^2 - 0.5t + 0.5$ rd; $\theta' = 4t - 0.5$

4)Determine the angular acceleration at any instant t

The angular acceleration is the derivative: $\theta'' = 4 \ \ \text{rd}/s^2$

5)Calculate the linear acceleration at $t_1 = 1s$

$$a_n = R.\theta'^2 = 2(3.5)^2$$

$$a_n = 24.5 \ m/s^2$$

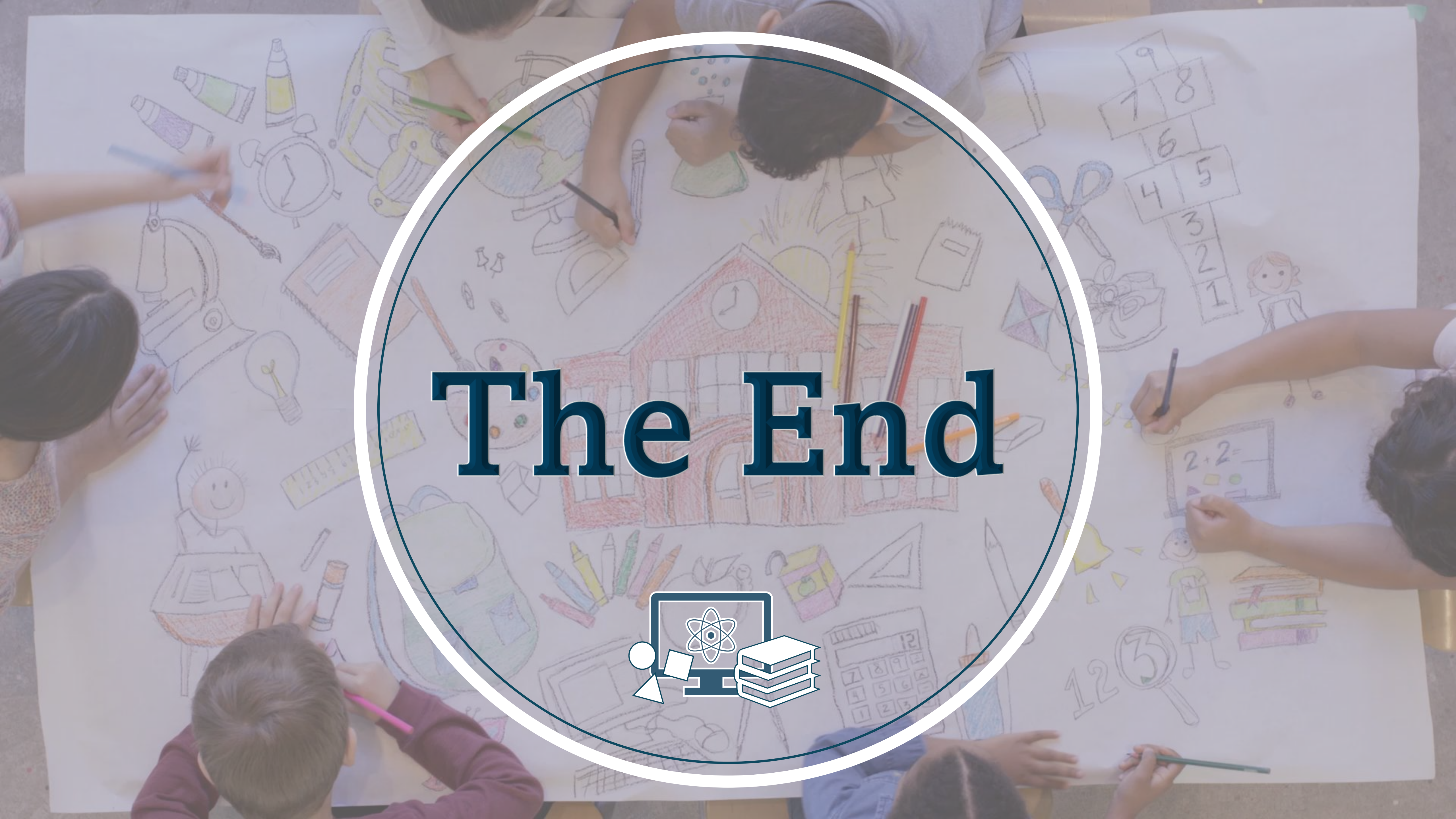
$$a_T = R.\theta'' = 2(4)$$

$$a_T = 8 \ \text{m}/s^2$$

$$a_1 = \sqrt{a_n^2 + a_T^2}$$

$$a_1 = \sqrt{24.5^2 + 8^2}$$

$$a_1 = 25.77 \ m/s^2$$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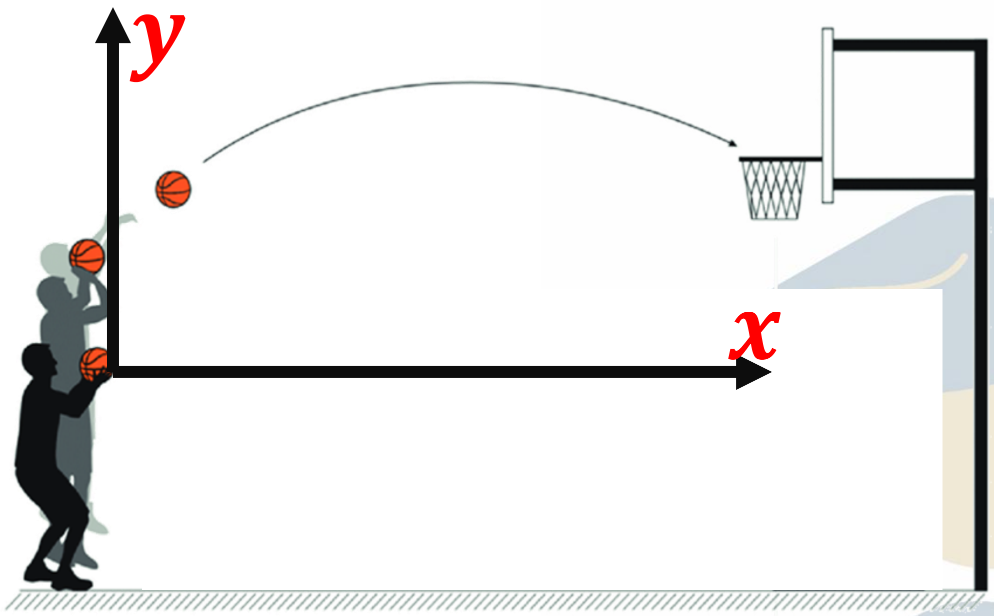


Physics - Grade 11 S

Unit Two: Mechanics

Chapter 7

Motion of a Particle in a Plane

Prepared & presented by : Mr. Mohamad Seif

Exercise 3:

The position vector of a moving particle M, in a reference (O, $\vec{i}, \vec{j}$), is given by: $\overrightarrow{OM} = \vec{r} = 2t.\vec{i} + \left(-4t^2 + 2t\right).\vec{j}$ in SI

1) **Determine the position vector of the particle $\vec{r}_0$ and $\vec{r}_2$ at the instants $t_0 = 0s$ and $t_2 = 2s$.**
2) **Determine the displacement vector of the particle (M) between the two instants.**
3) **Determine the equation of trajectory of the particle (M). Deduce the shape of the trajectory.**

Exercise 3:

$$\overrightarrow{OM} = \vec{r} = 2t.\vec{i} + \left(-4t^2 + 2t\right).\vec{j} \qquad \text{in SI}$$

1)Determine the position vector of the particle $\vec{r}_0$ and $\vec{r}_2$ at the instants $t_0 = 0s$ and $t_2 = 2s$.

$$\vec{r}_0 = 2(0).\vec{i} + \left(-4(0)^2 + 2(0)\right).\vec{j}$$

$$\vec{r}_0 = 0.\vec{i} + 0.\vec{j}$$

$$\vec{r}_2 = 2(2).\vec{i} + \left(-4(2)^2 + 2(2)\right).\vec{j}$$

$$\vec{r}_2 = 4.\vec{i} + (-16 + 4).\vec{j}$$

$$\vec{r}_2 = 4.\vec{i} - 12.\vec{j}$$

Exercise 3:

$$\overrightarrow{OM} = \vec{r} = 2t.\vec{i} + (-4t^2 + 2t).\vec{j} \quad \text{in SI}$$

2) Determine the displacement vector of the particle (M) between the two instants.

$$\Delta\vec{r} = \vec{r}_2 - \vec{r}_0$$

$$\Delta\vec{r} = 4.\vec{i} - 12.\vec{j} - [0.\vec{i} + 0.\vec{j}]$$

$$\Delta\vec{r} = 4.\vec{i} - 12.\vec{j}$$

Exercise 3:

$$\overrightarrow{OM} = \vec{r} = 2t.\vec{i} + \left(-4t^2 + 2t\right).\vec{j} \qquad \text{in SI}$$

3) Determine the equation of trajectory of the particle (M). Deduce the shape of the trajectory.

$x = 2t \Rightarrow t = \frac{x}{2}$

Substitute t in y.

$$y = -4t^2 + 2t$$

$$y = -4\left[\frac{x}{2}\right]^2 + 2\left[\frac{x}{2}\right]$$

$$y = -4\left[\frac{x^2}{4}\right] + 2\left[\frac{x}{2}\right]$$

$$y = -x^2 + x$$

The trajectory is parabola

Exercise 3:

4) Determine the average velocity vector between the two instants.

5) Determine the velocity vector at the instant t, then the value of the speed.

6) Determine the acceleration vector of the particle M. deduce its magnitude.

7) Determine the magnitude of the tangential acceleration of (M) at any instant t. deduce its value for t=2s

8) Calculate the value of normal acceleration at t=2s.

9) Deduce the radius of curvature at t=2s

Exercise 3:

4) **Determine the average velocity vector between the two instants.**

$$\vec{V}_{av} = \frac{\Delta\vec{r}}{\Delta t} \Rightarrow \vec{V}_{av} = \frac{\vec{r}_2 - \vec{r}_0}{t_2 - t_0}$$

$$\vec{V}_{av} = \frac{4.\vec{i} - 12.\vec{j} - [0.\vec{i} + 0.\vec{j}]}{2 - 0}$$

$$\vec{V}_{av} = \frac{4.\vec{i} - 12.\vec{j}}{2} \Rightarrow \vec{V}_{av} = 2.\vec{i} - 6.\vec{j}$$

Exercise 3:

$$\overrightarrow{OM} = \vec{r} = 2t.\vec{i} + \left(-4t^2 + 2t\right).\vec{j} \qquad \text{in SI}$$

5) Determine the velocity vector at the instant t, then the value of the speed.

$\vec{V}$ is the derivative of $\vec{r}$

$$\vec{V} = \vec{r}' = 2\vec{i} + (-8t + 2)\vec{j}$$

$$V = \sqrt{V_x^2 + V_y^2}$$

$$V = \sqrt{(2)^2 + (-8t + 2)^2}$$

$$V = \sqrt{4 + 64t^2 + 4 - 32t}$$

$$V = \sqrt{64t^2 - 32t + 8}$$

$$V = \sqrt{4(16t^2 - 8t + 2)}$$

$$V = 2\sqrt{16t^2 - 8t + 2}$$

Exercise 3:

$$\vec{V} = \vec{r}' = 2\vec{i} + (-8t + 2)\vec{j}$$

6) Determine the acceleration vector of the particle M. deduce its magnitude.

$\vec{a}$ is the derivative of $\vec{V}$: $\vec{a} = 0\vec{i} - 8\vec{j}$ ⇒ $\vec{a} = -8\vec{j}$

$a = \sqrt{a_x^2 + a_y^2}$ ⇒ $a = \sqrt{(0)^2 + (-8)^2}$

$$a = 8m / s^2$$

Exercise 3:

$$V = 2.\sqrt{16t^2 - 8t + 2}$$

7) Determine the magnitude of the tangential acceleration of (M) at any instant t. deduce its value for t=2s

a_t is derivative of of velocity value at any time

$$a_t = 2.\frac{32t - 8}{2\sqrt{16t^2 - 8t + 2}}$$

$$a_t = \frac{32t - 8}{\sqrt{16t^2 - 8t + 2}}$$

$$a_t = \frac{32(2) - 8}{\sqrt{16[2]^2 - 8[2] + 2}}$$

$$a_t = \frac{64 - 8}{\sqrt{64 - 16 + 2}}$$

$$a_t = \frac{56}{\sqrt{50}} \Rightarrow a_t = 7.91 m / s^2$$

Exercise 3:

8) Calculate the value of normal acceleration at t=2s.

$$a^2 = a_t^2 + a_n^2$$

$$a^2 - a_t^2 = a_n^2$$

$$[8]^2 - [7.91]^2 = a_n^2$$

$$64 - 62.56 = a_n^2$$

$$a_n^2 = 1.44$$

$$a_n = 1.2m / s^2$$

Exercise 3:

9) Deduce the radius of curvature at t=2s.

$$a_n = \frac{V^2}{R} \Rightarrow R = \frac{V^2}{a_n}$$

$$R = \frac{\left[2\sqrt{16t^2 - 8t + 2}\right]^2}{1.2}$$

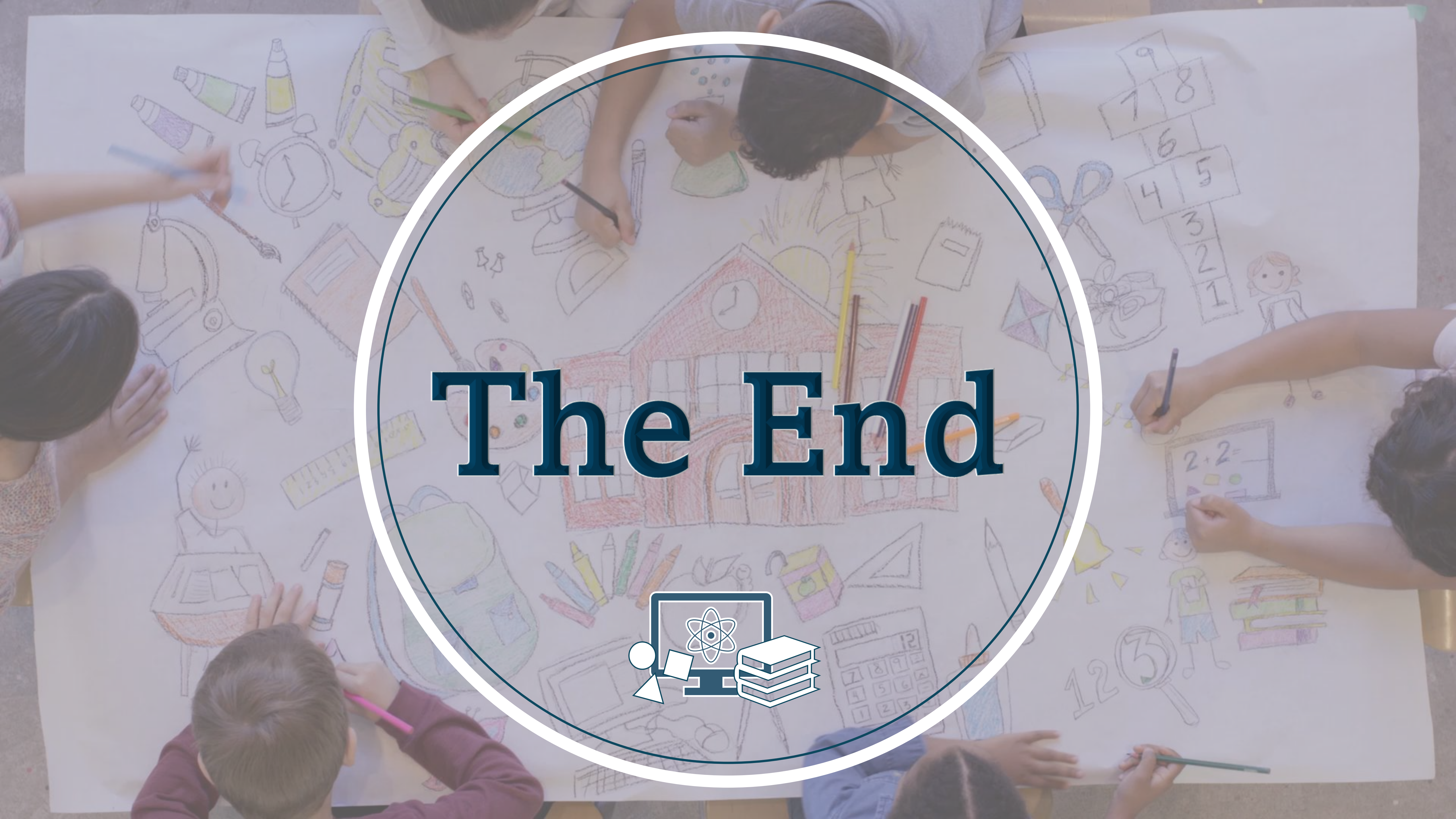
$$R = \frac{\left[2\sqrt{16t^2 - 8t + 2}\right]^2}{1.2}$$

$$R = \frac{\left[2\sqrt{16(2)^2 - 8(2) + 2}\right]^2}{1.2}$$

$$R = \frac{4[16(4) - 16 + 2]}{1.2}$$

$$R = \frac{4[64 - 16 + 2]}{1.2}$$

$$R = 166.6m$$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